Annotion

of dissertation by Shakir Aidos Ganizhanuly
for the degree of Doctor of Philosophy(PhD)
in the educational program "8D05401-Mathematics" on the topic:

"Inverse and direct problems for nonlinear Kelvin-Voigt equations"

One of the most dynamically developing areas of mathematics is the study of various problems in fluid mechanics. Currently, the development of mathematics, physics, mechanics, biology and the oil and gas industry, medicine, water resources and other fields of science and technology requires strict comprehensive mathematical research and ideological development of many Newtonian and non-Newtonian processes of fluid mechanics.

Purpose of the study. This dissertation is devoted to the study of problems of existence and uniqueness of strong and weak solutions of direct and inverse problems of a new formulation for linear and nonlinear Kelvin-Voigt (Navier-Stokes-Voigt) equations describing incompressible homogeneous and inhomogeneous fluid flows taking into account complex rheological properties. For the linear and nonlinear Kelvin-Voigt system, which describes the flow of non-Newtonian fluids taking into account all possible properties, the study of inverse and direct problems is important and relevant in both theoretical and practical terms.

Object of study. Inverse and direct problems for linear and nonlinear Kelvin-Voigt equations, describing the motion of an incompressible viscoelastic homogeneous and inhomogeneous fluid, and an inverse problem for a pseudoparabolic equation with p-Laplacian.

Research methods. The dissertation work used effective combinations of the following modern methods:
− Modern functional methods: the method of a priori estimates, the compactness method, the theory of Sobolev spaces, theorems of continuous and compact embedding, interpolation inequalities;
− Faedo-Galerkin method;
− monotonicity method;
− energy function method of functional analysis;
− general theory of direct and inverse problems;
− general theory of partial derivative equations.

Scientific novelty. Main results presented for defense. In the dissertation work, the statements of all solved problems and the results obtained are new, not previously studied. The following new results were obtained and presented for defense:
− The local existence and uniqueness of weak and strong solutions to the inverse problem for the nonlinear integro-differential Kelvin-Voigt system have been proven;

– The global existence and uniqueness of weak and strong solutions to the inverse problem for the linear integro-differential Kelvin-Voigt system have been proven;

– The local existence and uniqueness of weak and strong solutions to the inverse problem for the nonlinear integro-differential Kelvin-Voigt system with a special right-hand side have been proven;

– The existence and uniqueness of a weak and strong solution to the inverse problem for the Kelvin-Voigt integro-differential system, which is globally solvable, has been proven;

– The local existence and uniqueness of weak and strong solutions to the inverse problem for the nonlinear integro-differential Kelvin-Voigt system with a special integral overdetermination condition have been proven;

– Local and global existence and uniqueness of weak and strong solutions to the inverse problem for a pseudoparabolic equation with a p-Laplacian and a nonlinear source have been proven;

– Local and global existence and uniqueness of weak and strong solutions to the inverse problem for the pseudoparabolic equation with p-Laplacian and absorption are proven;

– The existence and uniqueness of a weak and strong solution to the initial boundary value problem for the Kelvin-Voigt equations, which describe the motion of an inhomogeneous viscoelastic fluid, have been proven.

Approbation. The results of the dissertation were reported and discussed at the conference "Problems of modern mathematics and its Applications" (Bishkek, Kyrgyzstan, June 16-19, 2021), "Non-local boundary value problems and related problems of mathematical biology, informatics and physics" (Nalchik , Kabardino-Balkaria, December 5-9, 2021), "Traditional international April at the conference" scientific conference in honor of the Day of Science Workers of the Republic of Kazakhstan "(Almaty, Kazakhstan, April 5-7, 2022 and 2023), "Functional Analysis in Interdisciplinary Applications" (Antalya, Turkey, October 2-7, 2023), as well as at scientific seminars of the Department of Mathematics of the Institute of Mathematics, Physics and Computer Science, Department of Mathematics and Mathematical Modeling of KazNPU named after Abay; Department of Mathematics, Faculty of Engineering and Natural Sciences of Suleyman Demirel University; Department of Mathematics, Faculty of Mechanics and Mathematics of Al-Farabi Kazakh National University.

Publications. Based on the results of the dissertation research, 12 works were published, including:

– 4 articles in scientific journals included in the first, second and third quartiles (Q1, Q2, Q3) of Clarivate Analytics Journal Citation Reports, respectively, and/or with CiteScore 99, 68, 56, 7 percentiles in the Scopus database, respectively:
1.Khompysh Kh., Kabidoldanova A., Shakir A. Inverse problems for nonlinear Navier-Stokes-Voigt system with memory//Chaos, solitons and fractals. – 177(12). – 2023. Web of Science: Q1, Scopus:99%.

2. Khompysh Kh., Shakir A. An inverse source problem for a nonlinear pseudoparabolic equation with p-Laplacian diffusion and damping term//Quaestiones Mathematicae. — 2022. — Vol. 46, no. 9. — P. 1889– 1914. Web of Science: Q3, Scopus:68%.

3. Khompysh Kh., Shakir A.G. Inverse problems for Kelvin-Voigt system with memory: global existence and uniqueness //Lobachevskii journal of mathematics. — 2023. — Vol. 44, no. 10. — Pp. 4341–4352. Web of Science: Q2, Scopus:56%.

4. Khompysh Kh., Shakir A.G. Time dependent inverse source problems for integro- differential Kelvin-Voigt system//Trends in Mathematics Series: Research Perspectives Ghent Analysis and PDE Center. —2023. Scopus:7%.

– 3 articles in journals recommended by the Committee for Quality Assurance in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan

1. Shakir A., Kabidoldanova A., Khompysh Kh. Solvability of a nonlinear inverse problem for a pseudoparabolic equation with p-Laplacian//Journal of Mathematics, Mechanics and Computer Science. — 2021. — Vol. 110, no. 2. — P. 35–46.

2. Shakir A. Blow-up of solutions the integro-differential Kelvin-Voigt equation//Bulletin of Physics and Mathematical Sciences. — 2022.—Vol. 79, no. 3. —P. 46–52.

3. Shakir A. Global solvability of inverse problem for linear Kelvin-Voigt equations with memory// Journal of Mathematics, Mechanics and Computer Science. — 2023. — Vol. 118, no. 2. — P. 30–41.

– 5 publications in the collection of abstracts of international conferences.

1. Khompysh Kh., Shakir A. Inverse problem for Kelvin-Voigt equations with memory//Materials of the conference:Inverse and ill-posed problems in natural sciences. — Almaty: 2023.—P. 23.

2. Khompysh Kh., Shakir A. Inverse problem for pseudoparabolic equations with p- Laplacian//Materials of the conference: Traditional international April scientific conference in honor of the Day of Science Workers of the Republic of Kazakhstan. — Almaty: 2021.—P. 91.

3. Khompysh Kh., Shakir A. An inverse problem for pseudoparabolic equations with p- Laplacian//Materials of the conference: Problems of modern mathematics and its applications. — Bishkek-Issyk-Kul: 2021. — P. 84.

4. Khompysh Kh., Shakir A. Inverse problem for Kelvin-Voigt equations with memory// Materials of the conference:Traditional international April scientific conference in honor of the Day of Science Workers of the Republic of Kazakhstan. — Almaty: 2023.—P. 138.

5. de Oliveira H.B., Khompysh Kh., Shakir A. Navier-Stokes-Voigt equations with non-negative density depentdent flows with vacuum//Materials of the

conference: Functional Analysis in Interdisciplinary Application. —Antalya: 2023.

Structure of the dissertation. The dissertation work consists of normative references, introduction, auxiliary results, four main sections (each section consists of subsections), conclusion and list of references.

Introduction includes the relevance of the research topic, goals and objectives, the main provisions for defending the dissertation, the object and subject of the research, the novelty and theoretical and practical significance of the research, the connection of the dissertation work with other research works, approbation of the work, publications of the author, volume and structure dissertations and contents.

In the preliminary results section introduces the necessary notation, gives the necessary definitions, well-known lemmas, continuous and compact embedding theorems and basic function spaces, as well as algebraic and functional inequalities.

In the first chapter considers the inverse problem for the system of integro-differential Kelvin-Voigt equations that describe the dynamics of a viscoelastic fluid. The applicability of the Faedo-Galerkin method is analyzed. The existence of a weak and a strong solution is proven. The conditions for the uniqueness of strong and weak solutions are studied. A priori upper estimates are obtained.

The second chapter is devoted to the study of the existence and uniqueness of weak and strong solutions of inverse problems for a system of Kelvin-Voigt integro-differential equations with a special integral redefinition condition. The local time solvability of the inverse problem in the weak and strong senses is proved. In addition, the global time solvability of the inverse problem under consideration is investigated.

In the third chapter considers the inverse problem for a pseudoparabolic equation with a p-Laplacian and a nonlinear term. Using the Faedo-Galerkin method, the existence of a weak solution was proven. Uniform a priori estimates for bounded above are obtained. Based on a priori data, the uniqueness of the solution is proven.

In the fourth chapter examines the initial boundary value problem for the system of Navier-Stokes-Voigt equations that describe the motion of a viscoelastic incompressible homogeneous fluid. The time-global existence, regularity and uniqueness of the initial-boundary value problem in the strong sense are proved.

The final section presents the conclusions and main results of the study obtained during the dissertation work.

Application area. The results obtained are used in solving linear and nonlinear direct and inverse problems of mathematics, hydrodynamics, and physics.

Degree of implementation. The results obtained have a wide range of theoretical and practical applications; they can be used and applied in solving linear and nonlinear applied problems in the fields of mathematics, hydrodynamics, physics, water, oil and gas industries, and others.

Candidate's contribution to the article. Based on the results obtained, 4 articles from journals included in the Scopus and Web of Science databases, 3 articles from domestic journals were published in the dissertation work. The methods and deep theoretical knowledge used in the articles were deeply understood and taken into account by the candidate. In addition, the results obtained in each section have a direct connection with the candidate. However, in some areas requiring theoretical understanding, the candidate sought advice from domestic and foreign leaders. The development of the article and the choice of journal were also analyzed by the candidate himself. The candidate has submitted manuscripts to scientific journals in six of the seven published articles and responded to suggestions and comments from reviewers.